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ABSTRACT

This report, a technical appendix to The Performance Contracting Concept in Education (EA 003 602), summarizes the mathematical models that have been developed to analyze contractual relationships and incentives. The report will be of interest primarily to theoreticians and analysts rather than educational administrators. (Author)

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THE PERFORMANCE CONTRACTING CONCEPT, APPENDIX: A CRITIQUE OF THE THEORY

James P. Stucker

A Report prepared for

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PREFACE

THIS STUDY is a technical appendix to J. P. Stucker and G. R. Hall, The Performance Contracting Concept, The Rand Corporation, R-699/1, May 1971. That Report discusses basic issues and considerations in using the performance contracting method of organizing educational programs, and is addressed to educational decisionmakers, particularly those guiding local school districts. This Appendix summarizes the mathematical models that have been developed to analyze contractual relationships and incentives, and will be of interest primarily to theoreticians and analysts rather than educational administrators.

Two further Rand reports on performance contracting are scheduled. One will analyze the operations and effects of some programs being conducted during the 1970-71 school year. The final report will be a Performance Contracting Guide, a general guide on how to plan, conduct and evaluate performance contracting programs. All three reports are part of Rand's study of performance contracting in education sponsored by the Assistant Secretary for Planning and Evaluation of the United States Department of Health, Education and Welfare pursuant to Contract No. HEW-OS-70-156.

SUMMARY

THIS TECHNICAL APPENDIX summarizes the major theoretical articles bearing on the theory of performance contracting. No general, definitive statement of that theory has been found, but most of the major elements of the theory are presented. Two especially pertinent contributions are reviewed: "A Formal Theory of the Employment Relationship," an article by H. A. Simon, and Optimal Rewards in Incentive Systems, a Ph.D. dissertation by G. M. Yowell, Jr. These analyses could provide a foundation for a fully articulated theory of performance contracting.

All the other articles reviewed are concerned with the theory of incentives for cost reduction. This theory, usually discussed in the context of the Federal Government and defense contractors, is simply a specialization of the general theory of incentives, and several of its assumptions are so restrictive that they severely limit its application to most performance contracting situations.

Simon investigates the possible tradeoffs in the choice between a sales contract (a contract for results) and an employment contract (a contract for labor resources). He points out that uncertainty is the major factor in these tradeoffs, and that the reduction of uncertainty achieved by delaying certain decisions is the major

advantage of an employment contract over a sales contract. His discussion can easily be rephrased to deal with the choice between a fixed contract, where all production decisions are made prior to signing the contract, and performance contracts, where some decisions may be deferred.

The theory of contracting is concerned with the choice between a sales contract and an employment contract. The theory of incentives is concerned with a choice that is applicable to either type of contract; its focus of interest is on methods for indirectly guiding the actions of the agent--a worker or contractor. The agent is not commanded to perform a specific action (or produce a specific product), but is given a range of choice. The employer (principal) attempts to influence the agent's choice by setting up a differential payment scale for the various actions that is biased in favor of the employer's (principal's) desires.

Yowell formulates a general decision-theoretic incentive model and investigates its properties under several sets of assumptions, including certainty and uncertainty. The incentive relationship he studies includes only two parties. This relationship is further simplified for purposes of analysis so that the payoff to the principal results exclusively from the action of the agent, while the agent's reward is determined solely by the principal according to the perceived results of the agent's actions. Yowell assumes that the basic relationship between the two parties is formed for the benefit of both, and investigates the extent to which the principal can guide the agent's actions by establishing rewards conditional on the results the agent achieves. Thus a prime characteristic of the approach is the free choice of action by the agent. The basic assumption that endows the principal with control over the agent's action is that the agent maximizes his profit (or, in the case of uncertainty, his expected utility). Rewards for results coupled with this profit-maximizing behavior allow the principal to influence the behavior of the agent without, at the same time, restricting the agent's free choice of action.

Yowell is able to prove the existence of optimal reward functions in most situations. More important, he shows that risk preferences are the critical factors in deriving reward functions, and that when the principal and the agent share a common belief concerning the joint probability of outcome and cost, the optimal reward function is a simple additive function of 1) the agent's cost and 2) a linear function of joint profits.

To our knowledge, however, the theory of contracting and the theory of incentives have never been integrated. Simon's work is based on the assumption that it may be advantageous to defer production decisions. The theory of incentives, although it is based on the agent's freedom of choice, concerns itself only with situations where he makes his production decisions early in the contract and, we must assume, never alters his plans. Thus, the only interesting aspect of the theory is the selection of an (optimal) reward function; and under these conditions, the only point of interest on the reward function is the point corresponding to the principal's preferred outcome.

If the theory of incentives were broadened to include the agent's response to unforeseen events, it would become much richer and more realistic, and could be used as a theory of performance contracting. In Section II of R-699/1, The Performance Contracting Concept, we attempt to synthesize these thoughts on the theory of contracting, the theory of incentives, and the integration of the two theories, into the outline of an informal statement of the theory of performance contracting.

CONTENTS

I. INTRODUCTION

A CONTRACT is a legal agreement between two parties in which one party promises to perform some specific act or acts in return for a consideration of value from the other. Every contract has the performance of a promise as its essence, and most contracts contain specific redress to cover nonperformance. There are two basic distinctions between types of contracts. The first is whether the contract calls for the provision of resources or the provision of results. The crucial distinction between these types of contracts is whether the principal party buys the resources and then directs their utilization to achieve some desired results, or whether he contracts directly for the results. The second distinction is whether the contract specifies a single acceptable outcome and a single acceptable payment, or a range of acceptable outcomes and payments. Contracts that allow for a range of acceptable outcomes and specify a range of payments corresponding to these outcomes are termed performance or incentive contracts.

Performance contracting is not a novel concept. For centuries people have been rewarded according to their performance--that is, according to the effort they expend or the results they produce. Our review of the literature, however, has produced no comprehensive theoretical statement concerning the benefits and costs of performance contracting as compared with other types of contracting. There are apparently no general models dealing with the tradeoff relationships between contracts for resources and contracts for results, and between fixed contracts, where all production decisions are made prior to signing the contract, and performance contracts, where some decisions may be deferred.

The literature does contain an important article, however, that provides a foundation for a theory of contracting by investigating the advantages and disadvantages of delayed decisionmaking. The literature also contains some refined statements concerning pricing or reward functions for use in a performance contract. In this technical appendix we review the main articles in these fields and suggest how they might be synthesized into a theory of performance contracting.

II. THE THEORY OF CONTRACTING

THE SALIENT ARTICLE on the theory of contracting is by H. A. Simon, who discusses the conditions under which two parties will find it mutually advantageous to enter into an employment contract rather than a sales contract.* In the terminology of the present Report, the choice is between contracting for resources and contracting for results.

Simon's approach is to discuss the authority relationship that exists between an employer B (for "boss") and an employee W (for "worker"). The collection of specific actions that W performs on the job (typing and filing letters, laying brick) are called his <u>behavior</u>. The set of all possible behavior patterns of W are considered, and x is used to designate an element of this set. A particular x might then represent a given set of tasks, performed at a particular rate of working, a particular level of accuracy, and so forth.

B is said to exercise <u>authority</u> over W if W permits B to select x. That is, W accepts authority when his behavior is determined by B's decision. In general, W will accept authority only if x^o, the

*H. A. Simon, "A Formal Theory of the Employment Relationship," <u>Econometrica</u>, Vol. 19, No. 3, July 1951, pp. 293-305.

x chosen by B, is restricted to some given subset (W's "area of acceptance") of all the possible values.

W enters into an employment contract with B when he agrees to accept B's authority and B agrees to pay W a stated wage (r). This form of contract differs fundamentally from a sales contract. In a sales contract each party promises a specific consideration in return for the consideration promised by the other. The buyer (like B) promises to pay a stated sum of money; but the seller (unlike W) promises a specified commodity in return. Moreover, the seller is not interested in the way in which his commodity is used once it is sold while the worker is interested in what the entrepreneur will want him to do (which x will be chosen by B).

> We notice that certain services are obtained by buyers in our society sometimes by a sales contract, sometimes by an employment contract. For example, if I want a new concrete sidewalk, I may contract for the sidewalk or I may employ a worker to construct it for me.*

Simon postulates that the employer and the employee are interested in maximizing their respective "satisfaction functions":

(1) $$S_b = S_b(x,r) = S_{b1}(x) + S_{b2}(r)$$

where

$$S_{b1}(x) \geq 0 \quad \text{and} \quad S_{b2}(r) \leq 0, \quad \text{and}$$

(2) $$S_w = S_w(x,r) = S_{w1}(x) + S_{w2}(r)$$

where

$$S_{w1}(x) \leq 0 \quad \text{and} \quad S_{w2}(r) \geq 0$$

The problem, then, is for the two parties to 1) agree upon a mutually acceptable outcome (x^o, r^o) and enter into a sales contract,

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*Ibid., p.294.

or 2) to agree on a wage (r^o) and enter into an employment contract whereby B is able to select unilaterally (at some later time) the action W must perform.

Simon then demonstrates that:

A. When the satisfactions to be derived from each possible action are known with certainty, the rational procedure for B and W is first to determine a preferred x, and then proceed to bargain about r so as to fix S_b and S_w; that is, they should arrive at a sales contract of the ordinary kind in which W agrees to perform a specific, determinate act x^o in return for an agreed upon price r^o.

B. When the satisfactions to be derived from each possible action are not known with certainty, and when both B and W act to maximize their expected satisfactions, the preferability of an employment contract over a sales contract increases 1) as the range of the expected satisfactions for the different actions decreases and 2) as the uncertainty of the satisfactions increases.*

Simon's results are derived from a very restrictive model, but they are intuitively appealing. It is unfortunate that no one has investigated these relationships in a more general context.

Simon's work has three important implications for performance contracting:

1) The basic issue in considering a performance contract for results is whether it is or is not preferable to a contract for resources.

2) The basic distinction between the two types of contracts is authority relationships.

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*Ibid., p. 301.

3) The preferred choice between the two basic contracts is, in part, a function of the uncertainty connected with the project.

III. THE GENERAL THEORY OF INCENTIVES

SIMON'S ARTICLE on contracting is concerned with the choice between a sales contract and an employment contract. The theory of incentives is concerned with a choice that is applicable to either type of contract; its focus of interest is on methods for indirectly guiding the actions of the agent--a worker or contractor. The agent is not commanded to perform a specific action (or produce a specific product), but is given a range of choice. The employer (principal) attempts to influence the agent's choice by setting up a differential payment scale for the various actions that is biased in favor of the employer's (principal's) desires.

By far the best general statement of the incentive problem is found in G. M. Yowell's Optimal Rewards in Incentive Systems.* We shall restate Yowell's model, the assumptions it requires, and the theorems he derives. This general framework will then be used in the following section to discuss the work on incentives for cost reduction.

Yowell formulates a general decision-theoretic incentive model and investigates its properties under several sets of assumptions,

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*G. M. Yowell, Jr., Optimal Rewards in Incentive Systems, EES Student Thesis Series, Department of Engineering Economic Systems, Stanford University, Stanford, Calif., March 1969.

including certainty and uncertainty. The incentive relationship he studies is the simplest possible, comprising only two parties; it could represent, for example, a manager and a subordinate, the government and a contractor, or society at large and the individual. For convenience, the first party is referred to as the principal and the second party as the agent. This relationship is simplified for purposes of analysis so that the payoff to the principal results exclusively from the action of the agent, while the agent's reward is determined solely by the principal according to the perceived results of the agent's action. It is further assumed that the basic relationship between the two parties is formed for the benefit of both. Yowell investigates the extent to which the principal can guide the agent's action by establishing rewards conditional on the results the agent achieves. Thus a prime characteristic of this approach is the agent's free choice of action. The basic assumption that endows the principal with control over the agent's action is the assumption that the agent maximizes his profit (or, in the case of uncertainty, his expected utility). Without this assumption the rewards established would be meaningless. Rewards for results coupled with this profit-maximizing behavior allow the principal to influence the agent's behavior without, at the same time, restricting the agent's free choice of action.

The basic elements of the problem are:

x an action

$x \epsilon X$ where X is the set of possible actions

h the payoff to the principal

q the inducement cost to the agent

r the conditional reward to the agent

$r \epsilon R$ where R is the set of possible rewards

π_1 the profit of the principal

(1) $$\pi_1 = h - r$$

π_2 the profit of the agent

(2) $$\pi_2 = r - q$$

π the joint profit

(3) $$\pi = \pi_1 + \pi_2 = h - q$$

and the following definitions hold:

Action: An irrevocable commitment of resources, such as effort, time, or money. "Action" is used interchangeably with the term "decision."

Cost: Cost is taken to mean inducement cost, where the inducement cost q of an action $x \epsilon X$ is defined as the amount of money required to induce the agent to take the action x, and may be equated with the opportunity cost of the action x, i.e., the loss of profit from other opportunities by taking the action x.

Incentive: A conditional reward, provided only if some stated result is achieved. "You will be rewarded with A if you accomplish B."

Conditional Reward: The conditional reward is a scalar function of the two variables, payoff h and cost q: $r(h,q)$. As a special case, the conditional reward may be defined in terms of only one of the two variables, h or q.

Thus, a particular action x^k will result in some particular, but perhaps unknown, outcomes h^k and q^k, which will define a unique reward r^k. (But note that the reward is based on the outcome, not on the action. This distinction is important when uncertainty is considered, in which case there will correspond, to each action, one or more possible outcomes and consequently one or more possible rewards.) We can now define:

Compensation: The agent is compensated for an action j if

$$r^j - q^j \geq 0 . \tag{4}$$

If the equality holds, then the agent is exactly compensated.

Weak Motivation: The agent is defined to be weakly motivated to take an action j in preference to all other actions k if the reward vector $r = (r^1, \ldots, r^n)$ is such that

$$r^j - q^j \geq r^k - q^k \quad \text{for all } k. \tag{5}$$

Strong Motivation: The agent is defined to be strongly motivated to take an action j if the reward vector r is such that the strict inequality holds in (5).

THE INCENTIVE PROBLEM UNDER CERTAINTY

In the case of certainty, the payoff h and the cost q will be related uniquely with each activity x. They will thus be expressed as

$$h = h(x) \quad \text{and} \tag{6}$$

$$q = q(x). \tag{7}$$

It is assumed that each party will seek to maximize his profit with the variables under his control. The principal can adjust the reward for each possible outcome of payoff and cost which the agent might produce, while the agent, in viewing the array of rewards established by the principal and his own costs, can select that action which maximizes his profit.

Yowell then assumes that a finite number, N, of alternative actions are available to the agent. Let $X = \{x^1, \ldots, x^N\}$ be the set of these actions x^k, $k=1, \ldots, N$. Under conditions of certainty, each action x^k will result in a particular outcome h^k and q^k. The problem of the principal is to specify the reward vector, $r = \{r^1 \ldots r^n\}$, where

$$(8) \qquad r^k = r(h^k, q^k) \quad k = 1, \ldots, N,$$

so as to maximize his profits.

How does the principal decide on the values of r^k? First, he will assume that he must provide a reward at least as great as the agent's cost; otherwise, the agent would receive a negative profit and would not accept the bargain. Thus for a desired action j,

$$(9) \qquad \pi_2 = r^j - q^j \geq 0.$$

Second, the principal knows that the agent is a profit-maximizer, and will take this knowledge into account by assuming that the agent will take whatever action maximizes his own profit. Hence the principal assumes that the agent will select the action j that maximizes $r^k - q^k$, i.e.,

(10) $$j \quad \max_k \; r^k - q^k .$$

Formally, Yowell states the principal's problem in deciding on the values of r^k as:

PROBLEM I. (i) $$\max_{(r^1, \ldots, r^n)\epsilon R} \{\pi_1 = h^j - r^j \mid \pi_2 = r^j - q^j \geq 0\}$$

where

(ii) $$j \quad \max_k \quad \{\pi_2 = r^k - q^k\}.$$

In words, Problem I states that the principal selects a set of rewards $(r^1, \ldots, r^n)$, corresponding to each payoff $((h^1,q^1), \ldots, (h^n,q^n))$, which maximizes his own profit through the action which the agent is motivated to take by the set of rewards. The constraint of I(i), which corresponds to (9), provides that the reward for the desired action j will be at least as great as the agent's cost, and the constraint I(ii), which corresponds to (10), accounts for the agent's profit-maximizing behavior. An additional constraint could be imposed--the requirement that the principal's profit be nonnegative--but this would add little to the results. Yowell then proves two theorems associated with Problem I.

Theorem I.1. There exists a reward vector $r = (r^1, \ldots, r^n)$ such that the agent can be motivated to select the action j corresponding to any desired outcome (h^j,q^j).

This reward vector, of course, has the properties

(11) $$r^j \geq q^j, \text{ and } r^k \leq q^k, \text{ for } k \neq j.$$

The solution to the principal's problem is, therefore, to pick the x^j that maximizes joint profits and to set up a reward vector with the properties

(12) $$r^j = q^j \text{ and } r^k < q^k \text{ for } k \neq j,$$

since his profit is maximized when the agent's profit is set at zero.

The next result applies to a linear reward function of the form $r = a + \alpha h$. Whereas Theorem I.1 guarantees that any outcome (h^j, q^j) can be motivated using a reward function such that (11) is satisfied, the next result guarantees only that the action which maximizes joint profit can be motivated when using a linear reward function.

Theorem I.2. Let j maximize the joint profit $\pi = \pi_1 + \pi_2 = h^k - q^k$, and let the reward function be linear with $r(h) = a + \alpha h$. The decision j will be motivated if and only if

(13) $$\alpha \geq \frac{q^j - q^k}{h^j - h^k} \quad \text{for} \quad h^j > h^k$$

and

(14) $$\alpha \leq \frac{q^j - q^k}{h^j - h^k} \quad \text{for} \quad h^j < h^k.$$

Furthermore, a value of $\alpha \leq 1$ can be found to satisfy (13) and (14). Yowell's proof also shows that $\alpha = 1$ will always satisfy (13) and (14).

This leads to a solution of the form,

$$r = q^j + \alpha[h^k - h^j], \tag{15}$$

where x^j is the joint profit-maximizing action and where x must satisfy (13) and (14). For future reference, note that the linear reward function is linear in the payoff to the principal and not the cost to the agent.

THE INCENTIVE PROBLEM UNDER UNCERTAINTY

Uncertainty is introduced into the incentive problem through the uncertain outcomes that result from a given action of the agent. In the problem under certainty, for each action x^k there was a unique payoff and cost outcome (h^k, q^k), while in the problem under uncertainty, for each action x there is a set of possible outcomes (h,q) which are assumed to be described by a probability distribution $f_x(h,q)$.

The simplicity of the incentive problem under certainty does not carry over into the uncertainty analysis, for the introduction of uncertainty requires the consideration of

1) the risk attitudes of the two parties,
2) the joint probability distribution on the outcomes, and
3) changes that occur in the state of knowledge of the parties during the transaction.

The utility analysis is taken mainly from Pratt.* If z is the number of dollars of income to a decisionmaker, then for any utility function U(z) the risk premium I is defined by

$$E[U(z)] = U(E[z] - I), \tag{16}$$

*J. W. Pratt, "Risk Aversion in the Small and in the Large," Econometrica, Vol. 32, Nos. 1-2, January - April 1964, pp. 122-136.

where E is the expectational operator. Given a utility function $U(z)$ and a probability distribution function $f(z)$, the expected values $E[U(z)]$ and $E[z]$ may be computed from

$$E[z] = \int_{-\infty}^{\infty} z\, f(z)\, dz. \tag{17}$$

Note that (16) defines the risk premium I implicitly by the function $U(z)$ and computed values $E[U(z)]$ and $E[z]$. To assure that I is determined uniquely, Yowell always assumes that $U(z)$ is monotonically increasing, and additionally that it is concave.

The risk premium I depends on both the utility function $U(z)$ and the probability distribution function $f(z)$. For the utility function $U(z)$, the risk tolerance function $\rho(z)$ is defined by

$$\rho(z) = -\frac{U'(z)}{U''(z)}, \tag{18}$$

where a prime indicates differentiation with respect to the argument z.

A risk-averse decisionmaker has a positive risk premium I, and a risk-preferring decisionmaker has a negative risk premium I. The assumption that $U(z)$ is monotonically increasing and concave implies that $\rho(z)$ is positive; that is, the decisionmaker is risk-averse and has a positive risk premium I. Two further definitions are required:

Cautiousness is defined as the change in the risk-tolerance function brought about by an increase in income, i.e., $\rho'(z)$.[*]

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[*] R. Wilson, On the Theory of Syndicates, Working Paper 71R, Graduate Business School, Stanford University, Stanford, California, August 1965; cited by Yowell, op. cit., p. 38.

Additionally, it will be useful to distinguish an alternative $\underline{x}$ as distinct if its associated probability distribution $f_{\underline{x}}(h,q)$ does not coincide with that of any other alternative $x \epsilon X$, i.e., there is no other alternative $x \epsilon X$ such that $f_{\underline{x}}(h,q) = f_x(h,q)$ for all h,q.

First Formulation: Identical Probability Beliefs on the Outcomes

The risk attitudes of the principal and the agent toward uncertain profit are described by their utility functions $U_1(\cdot)$ and $U_2(\cdot)$, where subscript 1 denotes the principal and 2 the agent. We assume that both parties will maximize their expected utility of profit.

The sequence of action within the problem consists of three parts: 1) the establishment of the reward function by the principal, 2) the action by the agent, and 3) the provision of the agreed-upon reward based upon the revealed outcome, which is a function of both the agent's action and the (uncertain) state of nature. The possible outcomes (h,q) that result from an action $x \epsilon X$ of the agent are understood to be random variables described by a joint probability distribution function $f_x(h,q)$, with the subscript referring to the action x. In this formulation, both parties are assumed to hold the same probability beliefs on the outcome, i.e., both parties agree to each of the distributions $f_x(h,q)$ $x \epsilon X$. The behavioral assumptions are that the principal will select, from the set R of allowable reward functions $r(h,q)$, that reward function which maximizes his expected utility, while the agent will select the decision $x \epsilon X$ that maximizes his expected utility. The decision x will in general depend upon the incentive function $r(h,q)$ selected by the principal.

Furthermore, we postulate for the agent an acceptance level A of expected utility, i.e., for the decision x selected by the agent, the expected utility must be greater than or equal to the acceptance level A; otherwise the agent would not enter into an agreement with the principal. Formally, these assumptions can be stated as:

PROBLEM II.

$$\text{(i)} \quad \max_{r \in R} \left\{ E_{\underline{x}} U_1(\pi_1) \,|\, E_{\underline{x}} U_2(\pi_2) \geq A \right\}$$

$$\text{(ii)} \quad \underline{x} \text{ maximize}_{x \in X} \; E_x U_2(\pi_2),$$

where E_x is the expectation operator with respect to the probability distribution $f_x(h,q)$, and R is the class of functions $r(h,q)$.

Note that the upper maximization is the principal's problem of selecting the optimal reward function, while the lower maximization is the agent's problem of selecting the optimal decision $\underline{x}$ as motivated by the reward function $r(\cdot)$. The principal's choice of an optimal reward function $r(\cdot)$ is made with the knowledge that the agent will react to any selected reward function in such a way as to maximize his (the agent's) expected utility.*

*Note that in the maximization of Problem II, for any monotonically increasing function $U(z)$, the maximization of $E[U(z)] = U(E[z] - I)$ from (16) may be accomplished by maximizing the argument $E[x] - I$. Hence we may restate the incentive problem under uncertainty as:

Problem IIa.

$$\text{(i)} \quad \max_{r \in R} \left\{ E_{\underline{x}} \pi_1 - I_1 \,|\, E_{\underline{x}} \pi_2 - I_2 \geq B \right\}$$

$$\text{(ii)} \quad \text{where } \underline{x} \; \max_{x \in X} \left\{ E_x \pi_2 - I_2 \right\}$$

If we restate the constraint II(ii) as

$$E_{\underline{x}} U_2(\pi_2) \geq E_x U_2(\pi_2) \qquad x \in X$$

Problem II may be written equivalently as

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where B is defined as

$$B \equiv U_2^{-1}(A).$$

Furthermore, if the constraint in IIa(i) is active,

$$E_{\underline{x}} \pi_2 - I_2 = B.$$

And

$$E_x \pi_1 = E_x h - E_x r$$

$$E_x \pi_2 = E_x r - E_x q$$

so that

$$E_x r = E_x q + I_2 + B$$

and

$$E_x \pi_1 - I_1 = E_x h - E_x q - I_1 - I_2 - B.$$

Then Problem IIa can be rewritten as

<u>Problem IIb.</u> (i) $\max_{r \in R} \left\{ E_{\underline{x}} h - E_{\underline{x}} q - I_1 - I_2 - B \right\}$,

where

$$\text{(ii)} \quad \underline{x} \in \max_{x \in X} \left\{ E_x r - E_x q - I_2 \right\}$$

Thus, the principal's problem in Problem II may be interpreted as the selection of a reward function that maximizes joint profits ($E_{\underline{x}} h - E_{\underline{x}} q$) minus the cost of uncertainty ($I_1 + I_2$).

Problem II.

$$\max_{r\epsilon R} \left\{E_{\underline{x}}U_1(\pi_1) \,|\, E_{\underline{x}}U_2(\pi_2) \geq A\right\}$$

subject to

$$E_{\underline{x}}U_2(\pi_2) \geq E_xU_2(\pi_2) \qquad x\epsilon X.$$

As now written, Problem II suggests a Lagrange multiplier solution; consequently, we define the Lagrange function L as

$$(19)\quad L(\underline{x},r,\lambda,\mu) = E_{\underline{x}}U_1(\pi_1) + \lambda[E_{\underline{x}}U_2(\pi_2)-A] + \sum_{x\epsilon X} \mu_x[E_{\underline{x}}U_2(\pi_2) - E_xU_2(\pi_2)]$$

where

$$\mu \equiv \{\mu_x : x\epsilon X\}$$

is the set of multipliers, which contains a multiplier μ_x for each alternative $x\epsilon X$. For convenience, it is assumed that the set X is finite, consisting of N alternatives.

To solve Problem II, we solve the equivalent saddle value problem of finding $\underline{x}^o$, r^o, λ^o, and $\mu^o \equiv \{\mu_x^o, x\epsilon X\}$ such that

$$(20)\qquad \min_{\lambda \geq 0,\ \mu \geq 0} \ \max_{\underline{x}\epsilon X,\ r\epsilon R} L(\underline{x},r,\lambda,\mu) = L(\underline{x}^o,r^o,\lambda^o,\mu^o),$$

where the notation $\mu \geq 0$ will denote $\mu_x \geq 0$ for $x\epsilon X$. Let us now characterize the action $\underline{x}^o$, which satisfies the saddle value problem (20), by the following theorem:

Theorem II.1. The action x^o which is preferred by the principal, and consequently satisfies (20), has

$$E_{\underline{x}^o}\left[U_1(\pi_1) + \left(\lambda^o + \sum_{x\epsilon X} \mu_x^o\right)U_2(\pi_2)\right] \geq \tag{21}$$

$$E_{\underline{x}}\left[U_1(\pi_1) + \left(\lambda^o + \sum_{x\epsilon X} \mu_x^o\right)U_2(\pi_2)\right]$$

$$\text{for } x\epsilon X \ .$$

Theorem II.1 states that the principal prefers the action $\underline{x}^o$ which maximizes an appropriately weighted sum of the expected utilities of the two parties.

It will be useful to single out those alternatives that play a part in the determination of r^o as "competing alternatives," for these alternatives are essentially in competition with the alternative desired by the principal.

> Competing Alternative. An alternative x is a competing alternative with respect to the alternative $\underline{x}^o$ preferred by the principal, if $\mu_x^o > 0$ in the solution of the saddle-value problem (20). If $\mu_x^o = 0$, then the alternative x is not competing.

To see the origin of the defined term, consider an alternative x that is not competing. From the theory of mathematical programming, the Lagrange multiplier associated with an inequality constraint must be zero if the constraint is not active, while the constraint must be active if the multiplier is positive.* Thus, if the constraint corresponding to the alternative x in (18) is not active, i.e., the equality does not hold in the constraint, then

$$E_{\underline{x}^o}U_2(r^o - q) > E_xU_2(r^o - q),$$

*H. W. Kuhn and A. W. Tucker, "Nonlinear Programming," in J. Neyman (ed.), Proceedings of the Second Berkeley Symposium on Mathematics, Statistics and Probabilities, University of California Press, Berkeley, 1950, pp. 481-492.

and the multiplier μ_x must be zero. Now, the inequality above states directly that the expected utility of the agent is greater for $\underline{x}^o$ than for x, so that the agent does not prefer x and x is thereby not a competing alternative.

Competing alternatives arise because of differences in the utility functions of the principal and the agent. This is illustrated by the following theorems of Yowell.

A. If there are no competing alternatives:

Theorem II.2. The optimal reward function is of the form $r(h,q) = q + g(\pi)$ for $f(h,q) > 0$, where q is the observed cost, and $g(\pi)$ is some function of joint profit π, with π defined as $\pi \equiv \pi_1 + \pi_2 = h - q$.

Theorem II.3. The function $g(\pi)$ is a strictly increasing function of π if both parties have positive risk tolerance.

Theorem II.4. The function $g(\pi)$ is linear in the joint payoff π if and only if the principal and the agent have identical cautiousness, i.e., $\rho'_1(\pi - \pi_2) = \rho'_2(\pi_2)$.

B. If there may be competing alternatives:

Theorem II.5. Assume there is uncertainty on the outcome (h,q) for a distinct preferred alternative $\underline{x}^o$. The optimal reward r^o is a function of the form

$$r(h,q) = q + g(\pi)$$

for

$$f_{x^o}(h,q) > 0$$

if and only if in solving the saddle-value problem (20),

$$\mu_x^0 = 0 \qquad x\epsilon X.$$

Theorem II.6. For any set of alternatives, the optimal reward for Problem II is identical to the optimal reward when there are no competing alternatives if and only if both parties have identical cautiousness.

An additional definition is required for the final theorem associated with Problem II.

Mutually Exclusive Outcome. The outcome h',q' for an alternative described by $f_x(h,q)$ is mutually exclusive with the outcomes (h,q) described by $f_{\underline{x}}(h,q)$ if $f_{\underline{x}}(h',q') = 0$.

Theorem II.7. If the possible outcomes of the desired alternative $\underline{x}^0$ and at least one outcome of each of the remaining alternatives $x\epsilon X$ are mutually exclusive, then there can be no competing alternatives, i.e., $\mu_x = 0 \;\; x\epsilon X$. Under these conditions the optimal reward function is obtained for the outcomes of the desired alternatives by specifying a reward function consistent with the principal's preferences, and for the mutually exclusive outcomes by penalizing at least one of these outcomes so as to induce in the agent a preference for the (principal's) desired alternative. To see that this is true, denote the desired alternative by $\underline{x}^0$ and the set of undesired alternatives by $\bar{X}$. Since there is at least one outcome for each of the alternatives in $\bar{X}$ that is mutually exclusive with the outcomes of the alternative $\underline{x}^0$, the reward function may be established independently over the undesired outcomes. For any reward function r^0 defined over the outcomes (h,q) of the desired alternative, it is possible to define r^o over the outcomes (h,q) of the remaining alternatives in such a way as to make

$$E_{\underline{x}^o}(r - q) > E_x(r - q) \qquad x \epsilon X$$

by imposing a sufficiently large penalty on at least one of the mutually exclusive outcomes of the remaining alternatives, and this implies that $\mu_x = 0$ for $x \epsilon X$.

As a consequence of Theorems II.2 and II.7, the optimal reward for the desired alternative $\underline{x}^o$ is of the form

$$r(h,q) = q + g(\pi) \quad \text{for } f_{\underline{x}^o}(h,q) > 0,$$

while for the outcomes of the undesired alternatives it is not defined uniquely, being required only to sufficiently penalize the undesired outcomes. As far as the theory goes, a sufficiently large penalty on at least one of the mutually exclusive outcomes for each alternative would be quite satisfactory for optimality.

Second Formulation: Different Probability Beliefs

The relationship between principal and agent that has been developed is clearly asymmetrical in the sense that the agent has complete control over the action while the principal has complete control over the reward function. Furthermore, it can be expected that the cost q would be determined largely by the agent on the basis of his value system, while the payoff h would be determined almost exclusively by the principal on the basis of his value system.

It is reasonable that the asymmetry of this relationship might result in differing probability beliefs on payoff and cost, and indeed it might be argued that in the real world differing beliefs are more likely than not.

Yowell states the problem of determining the optimal reward function when differing beliefs exist as

PROBLEM III. (i) $\max\limits_{r\epsilon R} \left\{E^1_{\underline{x}}U_1(\pi_1) \mid E^2_{\underline{x}}U_2(\pi_2) \geq A\right\}$

(ii) $\underline{x} \max\limits_{x\epsilon X} E^2_x U_2(\pi_2)$

where

$$E^i_x U_i(\pi_i) = \int_{-\infty}^{\infty}\int U_i(\pi_i) f^i_x(h,q)\; dh\; dq \qquad i = 1,2$$

with $f^1_x(h,q)$ the probability belief of the principal and $f^2_x(h,q)$ the probability belief of the agent for an action $x\epsilon X$. The superscript of the expectation operator denotes expectation with respect to the probability assessment of the indicated party, 1 or 2. R is the class of function $r(h,q)$.

For coincident probability beliefs, $f^1(h,q) = f^2(h,q)$ and we have the results given in Theorems II.2-II.4, which state that the optimal reward r^o is a function of the form

$$r(h,q) = q + g(\pi)$$

with $g(\pi)$ an increasing function of π, and that $g(\pi)$ is linear if and only if the two parties have identical cautiousness. For differing probability beliefs, $f^1(h,q) \neq f^2(h,q)$, and the earlier results no longer hold. The only result Yowell presents under these conditions is given by Theorem III.1.

Theorem III.1. The optimal reward function is a function of the form

$$r = q + g(\pi) + g_1(h,q) \text{ for } f^1(h,q) > 0 \text{ and } f^2(h,q) > 0$$

with $g(\pi)$ linear in π and independent of h,q, and $g_1(h,q)$ independent of π if and only if both parties have constant risk tolerances.

Comparing Theorem III.1 with Theorem II.4, we see that when probability beliefs differ, the special case of identical cautiousness with $\rho_1'(\pi_1) = \rho_2'(\pi_2) = 0$, i.e., constant risk tolerance, is required for a linear function $g(\pi)$. The function $g_1(h,q)$ will not be linear, in general, as it depends directly on the probability distributions $f^1(\cdot)$ and $f^2(\cdot)$.

Yowell's results concerning the form of the optimal reward function, while somewhat specialized, are nevertheless far more general than any other we have encountered. Section IV reviews the major articles on the theory of incentives for cost reduction, which is usually cited as the basis for most forms of government contracting. Yowell's general framework will be retained in order to place these models in perspective.

IV. THE THEORY OF INCENTIVES FOR COST REDUCTION

THE THEORY of incentives is concerned with the specification of a payoff vector that will induce the agent to select and accomplish actions that the principal prefers. Almost independently, a theory of incentives for cost reduction has been developed. This latter theory, usually expressed in terms of the Federal Government and defense contractors can, however, be viewed as a specialization of the former. This section reviews the major articles on the theory of cost incentives and contrasts them with each other and with Yowell's more general model.

Two features of the theory of incentives for cost reduction differentiate it immediately from Yowell's model. First, the assumption is always made that the payoff to the principal is fixed and given as a prerequisite of the problem. That is,

$$h = \underline{h}, \tag{22}$$

which implies that we are interested no longer in the joint distribution of outcomes $f_x(h,q)$, but only in the marginal distribution of costs $f_x(q|\underline{h})$. This assumption severely limits the applicability of this theory, as one of the prime characteristics of most performance contracting situations is the variability of the quality (performance) of the product.

Second, the form of the reward function is specified as

$$r = C_a + g(C_a), \tag{23}$$

where C_a is the actual (direct) costs incurred in performance of the contract (and may or may not be equal to q, the inducement cost), and where $g(C_a)$ is usually linear in C_a. Thus the form (and usually several parameters) of the reward function is specified and the problem is to select optimal values for the (remaining) parameters of the function.

SCHERER'S MODEL*

In Scherer's model:

A. The payoff to the principal is fixed and given, so that

$$U_1(\pi_1) = U_1(\underline{h} - r) \equiv D_1(r). \tag{24}$$

B. Although there is uncertainty concerning q, both parties base their decisions strictly upon the E(q). That is, $D_1(\cdot)$ and $U_2(\cdot)$ are linear in money.

C.

$$q = C_a + C_o, \tag{25}$$

where C_a represents the production cost of performing the contract in question and C_o the associated opportunity (or, in Scherer's terminology, user) costs.

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*F. M. Scherer, The Weapons Acquisition Process: Economic Incentives, Harvard University Press, Boston, Mass., 1964. F. M. Scherer, "The Theory of Contractual Incentives for Cost Reduction," Quarterly Journal of Economics, Vol. 78, No. 2, May 1964, pp. 257-280.

D. Opportunity costs are linked to direct costs by the relationship,

(26) $C_o = b_2(C_t - C_a) + b_3(C_t - C_a)^2$ with $b_2, b_3 > 0$

where C_t is a predetermined constant representing the target production costs. Thus C_o is uncertain only because of the uncertainty associated with C_a.

E. The form of the reward function is specified as

(27) $$r = C_a + \pi_{2t} + \alpha C_t - \alpha C_a,$$

where C_a and C_t are as defined above, π_{2t} represents the agent's allowed target profit, and α represents the agent's sharing rate in production costs above or below C_t.* α is restricted to $0 \leq \alpha \leq 1$.

F. The relationship between target profit and the sharing rate is given institutionally as

(28) $$\pi_{2t} = a_1 + a_2\alpha + a_3\alpha^2.$$

G. C_t and α are controlled by the principal.

The Agent's Problem

Under these specifications, the agent's expected profits are

(29) $$E\pi_2 = E(r - q) = Er(C_a) - EC_a - EC_o(C_a).$$

The agent's problem then is

*Observe that this reward function does not correspond to any derived by Yowell.

(30) $$\max_{x \in X} E[U_2(\pi_2)],$$

which, because of the linearity assumption on U_2, can be stated as

(31) $$\max_{x \in X} E(\pi_2).$$

In other words, the agent is assumed to select the productive methods that result in an $E(C_a)$ that maximizes his expected profits. Substituting (26), (27), and (28) into (30) and assuming that the resulting expression is twice differentiable, Scherer shows that the agent will select x^o such that

(32) $$EC_a^o = C_t + \frac{b_2 - \alpha}{2\, b_3}.$$

Three properties of this result are of interest. First, for any allowable (positive) values of b_2 and b_3, EC_a^o will reach its minimum value when α is set at its maximum value of 1.0. Any lower value of α will increase EC_a^o relative to C_t. (That is, a lower value of α will result in a reduced "cost underrun" or an increased "cost overrun.") Second, the higher the value of b_2 for any given values of α and b_3, the greater EC_a^o will be in relation to C_t. When b_2 is greater than 1.0, EC_a^o will be larger than C_t and an expected cost overrun will be optimal, since α cannot exceed 1.0. Third, even when $\alpha = 0$, it cannot be said that the agent has no incentive for cost reduction. He clearly has an incentive to limit EC_a to that value at which it equals $C_t + \frac{b_2}{2b_3}$.

The Principal's Problem

Scherer assumes that C_t is determined outside the model, so that the principal's problem is to select a sharing rate α that will minimize his expected expenditures on the contract. That is,

(33) $$\underset{0 \le \alpha \le 1}{\text{Min}} E[D_1(r)] \text{ s.t. } EC_a = EC_a^o$$

which can be expressed as

(34) $$\underset{0 \le \alpha \le 1}{\text{Min}} Er[EC_a^o(\alpha),\alpha],$$

the solution of which is

(35) $$\alpha^o = \frac{1 + b_2 - 2a_2b_3}{2 + 4a_3b_3} \quad \text{so long as } a_3 > -\frac{1}{2a_2}.$$

<u>Negotiations Over the Sharing Rate</u>

Scherer also investigates the case where the agent is able to negotiate (or at least express his preferences) over the value of the sharing rate (and hence over π_{2t} as $\pi_{2t} = \pi_{2t}(\alpha)$). The principal's preferred rate is given by (35). The agent's preferred sharing rate is determined by solving

(36) $$\underset{0 \le \alpha \le 1}{\max} E(\pi_2).$$

This yields

(37) $$\alpha_2^o = \frac{b_2 - 2a_2b_3}{1 + 4a_3b_3}$$

and will be equivalent to the government's preferred rate only if

(38) $$b_2 = 1 + 2b_3(a_2 + 2a_3).$$

Unfortunately, little can be sai concerning the preferred rate of either party or the relationship between them.

Risk and Its Allocation

It is often stated that in Scherer's model "risk" is allocated by the selection of the sharing rate α. This is true; but it is not correct to say that the principal (government) assumes all of the risk, or that the agent has no risk, when $\alpha = 0$. Observe that for $\alpha = 1$,

$$E[D_1(\pi_1)] = E[D_1(r)] = \underline{D}_1 \tag{39}$$

$$\text{as} \quad E(r) = EC_a + \pi_{2t} + C_t - E(C_a) = \pi_{2t} + C_t = \text{a constant}$$

and

$$E[U_2(\pi_2)] = E(\pi_2) \tag{40}$$

$$\text{as} \quad E(\pi_2) = C_t + \pi_{2t} - EC_a - EC_o.$$

That is, when $\alpha = 1$ the agent bears all of the risk. For $\alpha = 0$, however,

$$E[D_1(\pi_1)] = E[D_1(r)] \tag{41}$$

$$\text{as} \quad E(r) = \pi_{2t} + EC_a$$

and

$$E[U_2(\pi_2)] = E[\pi_2] \tag{42}$$

$$\text{as} \quad E(\pi_2) = \pi_{2t} + EC_a - EC_a - EC_o = \pi_{2t} + EC_o.$$

That is, when $\alpha = o$ the principal bears all of the risk associated with production costs, but the agent retains the risk associated with the user or opportunity costs.

It should also be noted that this allocation of risk is completely incidental to Scherer's analysis, since both parties are assumed to be indifferent to whatever risk is present and to ignore it in their decision. Furthermore, the amount of risk present is determined outside the model by the specification of $\underline{h}$ (which specifies $f_x(q|\underline{h})$), and the expected level of profits is partially determined outside the model by the specification of C_t. Hence, Scherer's model says very little about decisionmaking under uncertainty. In particular the model does not investigate either parties' preferences among

a) risk assumption and size of risk,

b) risk assumption and expected size of profits, or

c) size of risk and expected size of profits.

INTRILIGATOR'S MODEL

In 1964, Intriligator developed a model that incorporated uncertainty into the theory of incentives for cost reduction.** He focused directly on the problem of developing preferred sharing rates, ignoring the agent's production decisions. His model can be formulated as follows.

A. The payoff to the principal is fixed and given, so that

(43) $$U_1(\pi_1) = U_1(\underline{h} - r) \equiv V_1(r),$$

where $V_1(r)$ is the principal's utility function.

B. Both parties possess quadratic utility functions and both are risk averse. That is,

*M. D. Intriligator, "Optimal Incentive Contracts," unpublished paper, 1964.

(44) $$V_1(r) = v_1 + v_2 r + v_3 r^2$$
$$\text{with} \quad V_1'(r) = v_2 + 2v_3 r < 0$$
$$\text{and} \quad V_1''(r) = 2v_3 < 0,$$

and

(45) $$U_2(\pi_2) = U_2(r - q) = u_1 + u_2(r - q) + u_3(r - q)^2$$
$$\text{with} \quad U_2'(r - q) = u_2 + 2u_3(r - q) > 0$$
$$\text{and} \quad U_2''(r - q) = 2u_3 < 0.$$

C.

(46) $$q = C_a$$

where C_a is inducement cost and is uncertain. The marginal distribution of q is $f(q|\underline{h})$ and, since $V_1(\cdot)$ and $U_2(\cdot)$ are quadratic, this distribution can be represented by μ and σ^2.

D. The form of the reward function is specified as

(47) $$r = C_a + \pi_{2t} + \alpha C_t - \alpha C_a \quad 0 \leq \alpha \leq 1.$$

E. C_t and π_{2t} are specified outside the model.

<u>The Agent's Problem</u>

Under these conditions the agent's problem of selecting a preferred sharing rate can be stated as

$$\max_{0 \leq \alpha \leq 1} U[E_2(r - q)].$$

Using (45), (46), and (47) and designating the agent's probability beliefs on C_a as μ_2 and σ_2^2, the solution is

$$\alpha_2^o = \frac{(\mu_2 - C_t)\left(\frac{u_2}{2u_3} + \pi_{2t}\right)}{(\mu_2 - C_t)^2 + \sigma_2^2}. \tag{48}$$

This preferred sharing rate for the agent has the following properties:

1) The preferred sharing rate depends on target profits and costs, parameters of the contractor's utility function, and the mean and variance of the contractor's cost density function.

2) By the assumptions about the agent's utility function, the preferred sharing rate is positive if:

$$\mu_2 - C_t < 0;$$

that is, if target costs exceed expected costs (an expected cost under-run).

3) The preferred sharing rate is zero if target costs equal expected costs.

4) An increase in the agent's variance, reflecting increased uncertainty about costs, leads to a decrease in the preferred sharing rate.

5) An increase in target profits also leads to a decrease in the preferred sharing rate.

The Principal's Problem

The principal's problem of selecting the sharing rate that he prefers can be stated as

$$\max_{0 \le \alpha \le 1} V_1[E_1(r)].$$

Using (44), (46), and (47) and designating the principal's probability beliefs on C_a as μ_1 and σ_1^2, the solution is

$$\alpha_1^o = \frac{(\mu_1 - C_t)\left(\frac{v_2}{2v_3} + \pi_{2t} + \mu_1\right) + \sigma_1^2}{(\mu_1 - C_t)^2 + \sigma_1^2} \tag{49}$$

and has the following properties:

1) The principal's preferred sharing rate depends on target profits and costs, the parameters of the principal's utility function, and the mean and variance of the principal's probability density function.

2) The principal's preferred sharing rate is positive if

$$C_t - \mu_1 < \frac{\sigma_1^2}{\frac{v_2}{2v_3} + \pi_{2t} + \mu_1} \tag{50}$$

Since v_3 is assumed to be negative, the right-hand side of (50) is positive if v_2 is negative or if $\frac{v_2}{2v_3} < \pi_t + \mu_1$. If the right-hand side is positive, then target costs less than (the principal's) expected costs (an expected cost overrun from the principal's point of view) is sufficient but not necessary for a positive sharing rate.

3) The preferred sharing rate is 1.0 if target costs equal expected costs.

4) An increase in the principal's variance, reflecting increased uncertainty about costs, leads to a decrease (increase) in the preferred sharing rate if there is an expected cost overrun (underrun).

5) An increase in target profits leads to a decrease (increase) in the preferred sharing rate if there is an expected cost underrun (overrun).

Comparing (48) with (49) we can see that, in general, the agent's preferred rate will differ from the principal's preferred rate. The actual sharing rate incorporated into a contract might be expected to fall somewhere between these two rates, and to depend on the relative power and bargaining skills of the two parties.

Identical Probability Beliefs

In the special case of identical probability beliefs, that is, where

(51) $$\mu_1 = \mu_2 = \mu \quad \text{and}$$

(52) $$\sigma_1^2 = \sigma_2^2 = \sigma^2,$$

the two preferred sharing rates will be related by

(53) $$\alpha_2^o \gtrless \alpha_1^o \quad \text{as} \quad C_t - \mu \lessgtr \frac{\sigma^2}{\mu + \frac{v_2}{2v_3} - \frac{u_2}{2u_3}} .$$

Now, $v_3 < 0$, $u_3 < 0$, and u_2 is necessarily positive, so again the rightmost expression is positive if $v_2 < 0$. Assuming this is true, target costs must exceed expected costs (an expected cost underrun) for the preferred rates to be equal. A larger expected underrun would result in the agent's preferred rate being greater than the principal's preferred rate. A small expected underrun or an expected overrun ($C_t - \mu < 0$) would lead to $\alpha_2 < \alpha_1$.

BERHOLD'S MODEL

Berhold expanded the treatment of uncertainty in 1967.* He did not discuss Intriligator's problem of negotiations over the sharing rate, but returned to Scherer's basic formulation, where the principal selects the sharing rate from a knowledge of the agent's profit-maximizing behavior.

In Berhold's model:

A. The payoff to the principal is fixed and given, so that

(54) $$U_1(\pi_1) = V_1(r).$$

B. The cost to the agent is composed of actual costs C_a and opportunity costs C_o, so that

(55) $$q = C_a + C_o.$$

C. The reward function is of the form,

(56) $$r = C_a + \pi_{2t} + \alpha C_t - \alpha C_a \quad \text{with} \quad 0 \le \alpha \le 1,$$

*M. Berhold, An Analysis of Contractual Incentives, Working Paper 129, Western Management Science Institute, University of California, Los Angeles, September 1967.

so that

$$(57)\qquad \begin{aligned}\pi_2 &= r - q\\ &= r - C_a - C_o\\ &= \pi_{2t} + \alpha C_t - \alpha C_a - C_o.\end{aligned}$$

Berhold assumes that the principal selects values for C_t, π_{2t}, and α at some point in time t_1. At some later time, t_2, the agent selects an action x and, hence, a level of (expected) cost q. Both parties seek to maximize their respective utility. He investigates the model under conditions of certainty at both t_1 and t_2, under conditions of uncertainty at t_1 and certainty and t_2, and under conditions of uncertainty at both t_1 and t_2. He also investigates the model under a variety of utility assumptions for the principal and the agent. Throughout Berhold's work the emphasis is upon the derivation of the principal's preferred sharing rate α^o. In its most general formulation, his problem can be expressed as the principal's problem:

$$(58)\qquad \max_{C_t,\ \pi_{2t},\ \alpha} \{E_1[U_1(\pi_1)] \mid E_2[U_2(\pi_2)] \geq A\}$$

subject to (the agent's problem):

$$(59)\qquad \max_{x \varepsilon X} E_2[U_2(\pi_2)].$$

His procedure is to solve (59) for x^o, and then to solve

$$(60)\qquad \max_{0 \leq \alpha \leq 1} E_1[U_1(\pi_1)]$$

subject to the restriction that

(61) $$E_2[U_2(\pi_2)] = A.^*$$

Under these conditions, Berhold derives the preferred sharing rates displayed in Table 1.

Table 1

BERHOLD'S OPTIMAL SHARING RATES

Case and Utility Function	Certainty at both t_1 and t_2	Uncertainty at t_1, certainty at t_2	Uncertainty at both t_1 and t_2
U_1 linear U_2 linear	$\alpha^o = 1$	$\alpha^o = 1$	$\alpha^o = 1$
U_1 strictly concave U_2 linear	$\alpha^o = 1$	$\alpha^o = 1$	$\alpha^o = 1$
U_1 linear U_2 strictly concave	$\alpha^o = 1$	$0 < \alpha^o < 1$	$0 < \alpha^o < 1$
U_1 strictly concave U_2 strictly concave	$\alpha^o = 1$	$0 < \alpha^o < 1$	$0 < \alpha^o < 1$

SOURCE: Ibid., p. 66.

In short, the principal prefers a sharing rate (for the agent) of 1.0, unless there is uncertainty and the agent is risk-averse. In the

*This implies that joint expected utility minus any costs of uncertainty is maximized, and then the agent's expected utility is set at the lowest acceptable value so as to maximize the principal's expected utility. Note also that (61) may be stated as

$$E[U_2(\pi_{2t} + \alpha C_t - \alpha C_a - C_o)] = A,$$

which provides one relation between α^o, π^o_{2t}, and C^o_t. Hence, if either π^o_{2t} or C^o_t is selected arbitrarily, the other may be determined.

latter cases, the principal obtains a better overall deal if he is willing to assume some of the risk, that is, $\alpha^{o} < 1.0$.

OTHER RELATED MODELS

Midler's Model

Midler has recently generalized the basic model of incentives for cost reduction.* He bypasses the agent's production decision and formulates a concave N-person game in which C_t, π_{2t}, and α are jointly determined by the preferences of the principal and one or more prospective agents through an "as if" bargaining procedure. Given the necessary information for the basic model as well as information on the relative bargaining strengths of the parties, he claims the equilibrium values for C_t, π_{2t}, and α can be obtained. Unfortunately, he provides us with no characterizations of these solution values.

McCall's Model

McCall has more recently shown an interesting result with another variation of the cost incentive model.** In his formulation, the reward function is

(62) $$r = C_a + \pi_{2t} + \alpha(C_t - C_a) \quad \text{with}$$

(62) $$\pi_{2t} = \beta\, C_t \quad \text{so that}$$

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*J. L. Midler, Optimal Incentive Contracting: A Constrained Game Theory Model, The RAND Corporation, P-4404, June 1970.

**J. J. McCall, "The Simple Economics of Incentive Contracting," American Economic Review, Vol. 60, No. 5, December 1970, pp. 837-846.

(63) $$r = (\alpha + \beta)\, C_t + (1 - \alpha)\, C_a.$$

The principal selects the sharing rate α and the target profit rate β and solicits bids from agents on the target cost C_t. It is assumed that there is a large number of potential bidders, so that each potential agent submits a bid equal to his costs of production plus his minimum acceptable profit. (Since McCall shows that his major conclusion also holds under conditions of uncertainty, we discuss only the certainty case.) If he submits a higher bid, he knows he has no chance of receiving the contract, and he will not submit a lower bid since it would likely result in a loss of profit.

Furthermore, McCall assumes that the product being bid upon also has an established market price. Agents that are selling the product in the free market have, from the profit-maximizing assumptions, costs of production equal to or less than the market price. These agents will not sell to the principal for a price less than the market price P, and they cannot expect to receive more. Thus, for these firms,

(64) $$r = (\alpha + \beta)\, C_t + (1 - \alpha)\, C_a = P \quad \text{for } C_a \le P.$$

Their bids will depend upon their costs of production and can be determined by solving (64) for C_t, which yields

(65) $$C_t^o = \frac{P - (1 - \alpha)\, C_a}{\alpha + \beta} \quad \text{for } C_a \le P.$$

Agents that are not selling in the free market but are capable of producing the product and are interested in submitting a bid to the principal must have, from the competitive assumptions, production costs higher than the market price. They are not selling at the market price because they would incur a loss. For the same reason, they will not sumbit a bid equal to the market price. The alternative profit for these firms is zero. Hence, if they bid on the contract, they will submit a bid incorporating a profit of, or very close to, zero. These agents (McCall calls them inefficient producers) will accept a lower profit than the other agents, but they have higher costs of production. Their bids will depend on their costs of production and can be derived by solving

(66) $$\pi_2 = r - C_a = (\alpha + \beta)\ C_t - \alpha C_a = 0$$

for C_t. This yields

(67) $$C_t^o = \frac{\alpha}{\alpha + \beta} C_a \qquad \text{for } C_a > P.$$

McCall then demonstrates that 1) if the principal receives bids from both groups of firms and 2) selects an agent solely on the basis of the lowest bid target costs, the principal <u>may</u> not be minimizing his own outlay (reward). This is illustrated in Fig. 1, where reward and target cost bids are plotted against production costs. Both axes represent dollars, and market price P can be shown on both. We have assumed that $\alpha = 0.80$ and $\beta = 0.10$. Target cost bids are computed using (65) for $C_a \leq P$ and (67) for $C_a > P$. Rewards are computed using (62).

It can be seen from Fig. 1 that if the principal received a full array of bids and selected the lowest (C_t^o) he would be minimizing his outlay with a reward equal to P. However, if less than a full

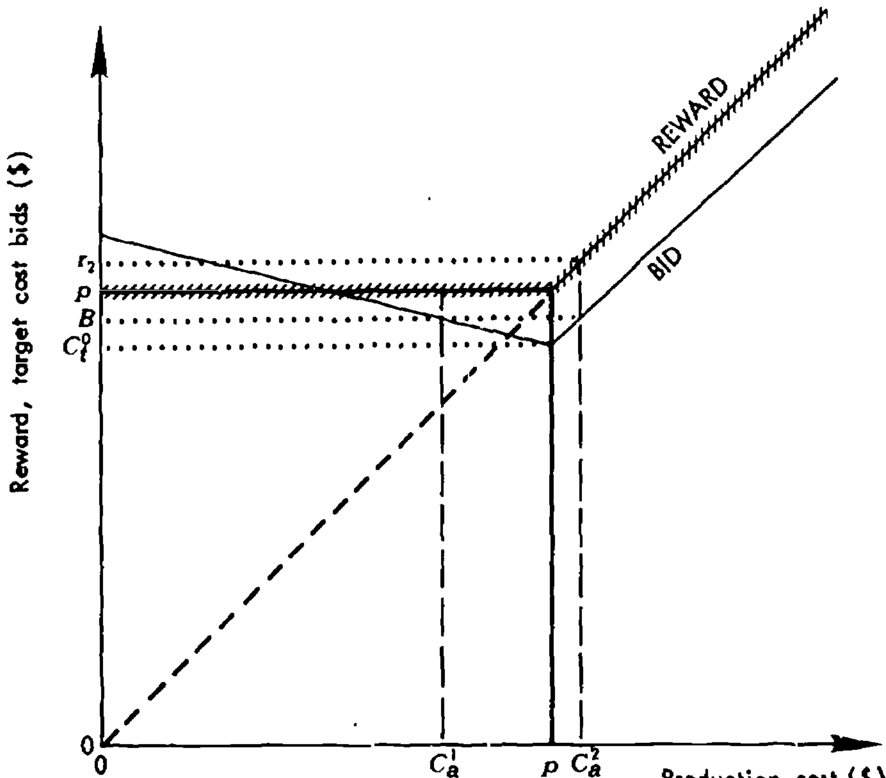

Fig. 1 - Bids and Rewards as Functions of Production Costs
(α = 0.8, β = 0.1)

array of bids is received, the outcome is not so sure. For example, if only two agents, with production costs of C_a^1 and C_a^2, submit bids, the bids will be identical and the principal may be indifferent between them. But the principal's payment will differ significantly depending upon which bid he accepts. If he selects the bid associated with C_a^1, the payment will be only P, whereas if he selects the bid associated with C_a^2, the payment will be r_2 (> P).

In short, McCall has shown that if there is an established market for a product, a principal soliciting cost-based bids can expect to pay no less than the market price and may pay substantially more.

V. SUMMARY AND CONCLUSIONS

THIS APPENDIX has summarized the major theoretical articles bearing on the theory of performance contracting. No general, definitive statement of that theory has been found but we believe that most of the major elements of the theory have been presented. Simon investigated the tradeoff involved in the choice between a sales contract and an employment contract. He showed that uncertainty is a major factor in these tradeoffs, and that the reduction of uncertainty achieved from delaying certain decisions is the major benefit of choosing an employment contract over a sales contract. His discussion can easily be rephrased to deal with the choice between a fixed contract and a performance contract. In fact, this transformation makes his arguments more meaningful and more intuitively appealing.

All the other articles reviewed here have dealt with some variation of the incentive problem, and all provide useful insights into various facets of the interactions between a principal and an agent. Many of these insights are directly applicable to performance contracting. For example, Yowell's statement of the reward (pricing) problem under conditions of uncertainty illustrates that the risk attitudes of both parties must always be considered, and that the buyer cannot simply set up the best possible deal for himself and expect the seller to respond as he (the buyer) wishes. And McCall's work on competitive bidding illustrates one of the pitfalls of cost-based pricing and the difficulty of competitive source-selection for performance contracting.

The theory of contracting and the theory of incentives, however, have never to our knowledge been integrated. Simon's work is based on the assumption that it may be advantageous to defer production decisions. The theory of incentives, although it is based on the agent's freedom of choice, concerns itself only with situations where he makes his production decisions early in the contract and, we must assume, never alters his plans. Thus, the only interesting aspect of the theory is the selection of an (optimal) reward function; and under these conditions, the only point of interest on the reward function is the point corresponding to the principal's preferred outcome.

If the theory of incentives were broadened to include the agent's response to unforeseen events, however, it would become a much richer and more realistic theory and, we believe, would come close to constituting a theory of performance contracting.

In such a theory, the nature of the reward function would be significantly altered. As before, the agent would view the reward function as his price function. As time passes and his knowledge of the world increases, however, he would probably alter his production techniques in a continuing attempt to maximize his profits given the price function and the new state of nature.

The perspective of the principal would also change. He would no longer view the reward function as simply a means of motivating the agent to produce <u>the</u> preferred output. He would realize that the uncertainties of production might result in the agent's viewing <u>any</u> point on the reward function as his profit-maximizing point. Hence, the principal would be concerned with the entire range of the reward function and it would, in fact, represent his tradeoff (indifference) function between output and reward.

Section II of R-699/1, <u>The Performance Contracting Concept</u>, attempts to synthesize thoughts on the theory of contracting, the theory of incentives, and the integration of the two theories into the outline of an informal statement of the theory of performance contracting.